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On the coherent states for the q-Hermite polynomials and related Fourier transformation

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Abstract. We discuss the Fourier–Gauss transformation properties of the continuous q-Hermite polynomials and associated q-coherent states.

One realization of the q-harmonic oscillator [1, 2] can be built on the finite interval $x \in [-1, 1]$ in terms of the continuous q-Hermite polynomials $H_n(x|q)$ with the parameter $q \in (0, 1)$ [3]. The factorization of the difference equation for $H_n(x|q)$ yields explicit lowering $b(x|q)$ and raising $b^+(x|q)$ operators, which satisfy the q-Heisenberg commutation relation [4]. As in the case of the well known non-relativistic quantum-mechanical oscillator, one can also construct coherent states for this q-deformed system. They are defined as eigenfunctions of the lowering operator $b(x|q)$ and involve the infinite series of the form [4]

$$I(x, t; q) = \sum_{n=0}^{\infty} \frac{q^{n^2/4} t^n}{(q; q)_n} H_n(x|q) \tag{1}$$

where $(a; q)_0 = 1$ and $(a; q)_n = \prod_{j=0}^{n-1}(1 - aq^j)$, $n = 1, 2, 3, \ldots$, is the q-shifted factorial. In the limit case when the parameter $q \equiv \exp(-2\kappa^2)$ tends to 1 (and, consequently, $\kappa \to 0$), we have

$$\lim_{q\to 1^-} \kappa^{-n} H_n(\sin \kappa s|q) = H_n(s) \tag{2}$$

where $H_n(s)$ are the classical Hermite polynomials. Therefore

$$\lim_{q\to 1^-} I(\sin \kappa s, 2\kappa\tau; q) = \sum_{n=0}^{\infty} \frac{\tau^n}{n!} H_n(s) = \mathrm{e}^{2s\tau - \tau^2} \tag{3}$$

and we recover the coherent states for the non-relativistic linear oscillator, according to the generating function in (3). Observe also that a summand in the Rogers generating function for the continuous q-Hermite polynomials [5]

$$\sum_{n=0}^{\infty} \frac{t^n}{(q; q)_n} H_n(\cos\theta|q) = e_q(t\mathrm{e}^{\mathrm{i}\theta}) e_q(t\mathrm{e}^{-\mathrm{i}\theta}) \qquad |t| < 1 \tag{4}$$

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falls short by the factor $q^{n^2/4}$ in order to express $I(x,t;q)$ in terms of the q-exponential function

$$e_q(z) := \sum_{n=0}^{\infty} \frac{z^n}{(q;q)_n} = (z;q)_\infty^{-1}. \tag{5}$$

Actually, as suggested in [6] (see also [7, 8]), series (1) can be used to define yet another q-exponential function $\varepsilon_q(x;\tau)$ (see (27) below), which differs from (5) and its reciprocal

$$E_q(z) := \sum_{n=0}^{\infty} \frac{z^n}{(q;q)_n} q^{n(n-1)/2} = (-z;q)_\infty. \tag{5'}$$

The advantage of the alternate q-exponential function $\varepsilon_q(x;\tau)$ is that it represents an eigenfunction of the Askey–Wilson divided difference operator $\mathcal{D}_q$. The standard q-analogues of the exponential function $e_q(z)$ and $E_q(z)$ do not possess this property.

The function $\varepsilon_q(x;\tau)$ is also expressible as a sum of two ${}_2\phi_1$ basic hypergeometric series (cf [6]). This sum will be evaluated by the aid of q-analogues of the well known representations

$$H_{2n}(x) = (-4)^n n! L_n^{(-\frac{1}{2})}(x^2) = (-4)^n (\tfrac{1}{2})_n \, {}_1F_1(-n;\tfrac{1}{2};x^2) \tag{6a}$$

$$H_{2n+1}(x) = (-4)^n n! 2x L_n^{(\frac{1}{2})}(x^2) = (-4)^n (\tfrac{3}{2})_n 2x \, {}_1F_1(-n;\tfrac{3}{2};x^2) \tag{6b}$$

for the Hermite polynomials $H_n(x)$ in terms of the Laguerre polynomials $L_n^{(\alpha)}(z)$ and the confluent hypergeometric function ${}_1F_1(a;b;z)$.

We start with the observation that the q-Hermite polynomials $H_n(x|q)$ are a particular case of the continuous q-ultraspherical polynomials of Rogers $C_n(x;\beta|q)$ with vanishing parameter β, that is

$$H_n(x|q) = (q;q)_n \, C_n(x;0|q). \tag{7}$$

On the other hand, the polynomials $C_n(x;\beta|q)$ are related to the continuous q-Jacobi polynomials

$$P_n^{(\alpha,\beta)}(x;q) = \frac{(q^{\alpha+1}, -q^{\beta+1};q)_n}{(q^2;q^2)_n} \, {}_4\phi_3\left(\begin{matrix} q^{-n}, q^{n+\alpha+\beta+1}, q^{\frac{1}{2}}\mathrm{e}^{\mathrm{i}\theta}, q^{\frac{1}{2}}\mathrm{e}^{-\mathrm{i}\theta} \\ q^{\alpha+1}, -q^{\beta+1}, -q \end{matrix} \middle| q;q \right) \tag{8}$$

by the quadratic transformations

$$C_{2n}(x;q^\lambda|q) = \frac{(q^\lambda, -q;q)_n}{(q;q^2)_n} q^{-n/2} P_n^{(\lambda-\frac{1}{2},-\frac{1}{2})}(2x^2-1;q) \tag{9a}$$

$$C_{2n+1}(x;q^\lambda|q) = \frac{(q^\lambda, -1;q)_{n+1}}{(q;q^2)_{n+1}} q^{-n/2} x P_n^{(\lambda-\frac{1}{2},\frac{1}{2})}(2x^2-1;q) \tag{9b}$$

in the variable $x = \cos\theta$ (see [9], formulae (7.5.35) and (7.5.36)). The basic hypergeometric series ${}_4\phi_3$ in (8) is a particular case of the definition

$${}_{n+1}\phi_n(a_1,\ldots,a_{n+1};b_1,\ldots b_n;q,z) = \sum_{k=0}^{\infty} \frac{(a_1,\ldots a_{n+1};q)_k}{b_1,\ldots,b_n,q;q)_k} z^k \tag{10}$$

with $n = 3$ and $(a_1,\ldots,a_n;q)_k = \prod_{j=1}^n (a_j;q)_k$ is the product of q-shifted factorials. Since

$$\lim_{q\to 1^-} C_n(x;q^\lambda|q) = C_n^\lambda(x) \qquad \lim_{q\to 1^-} P_n^{(\alpha,\beta)}(x;q) = P_n^{(\alpha,\beta)}(x) \tag{11}$$

the quadratic transformations (9) are q-analogues of the relations

$$C_{2n}^\lambda(x) = \frac{(\lambda)_n}{(\frac{1}{2})_n} P_n^{(\lambda-\frac{1}{2},-\frac{1}{2})}(2x^2-1) \qquad C_{2n+1}^\lambda(x) = \frac{(\lambda)_{n+1}}{(\frac{1}{2})_{n+1}} x P_n^{(\lambda-\frac{1}{2},\frac{1}{2})}(2x^2-1) \tag{12}$$

between the ultraspherical polynomials of Gegenbauer $C_n^{\lambda}(x)$ and the Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$.

Now we can use formulae (8) and (9a) in order to express $C_{2n}(x;q^{\lambda}|q)$ in terms of the basic hypergeometric series ${}_4\phi_3$. The subsequent passage to the limit $\lambda\to\infty$ will give the desired representation for $H_{2n}(x|q)$. Observe that explicit dependence on the variable $x=\cos\theta$ enters the q-Jacobi polynomials via the q-shifted factorials

$$(q^{\frac{1}{2}}\mathrm{e}^{\mathrm{i}\theta},q^{\frac{1}{2}}\mathrm{e}^{-\mathrm{i}\theta};q)_k=\prod_{j=0}^{k-1}(1+q^{2j+1}-2xq^{j+\frac{1}{2}})\qquad k=1,2,3,\ldots. \tag{13}$$

Therefore the substitution $\theta\to 2\theta$ in the relation (8) leads to polynomials with respect to the variable $\cos 2\theta=2\cos^2\theta-1=2x^2-1$. The polynomials $P_n^{\alpha,\beta)}(2x^2-1;q)$ are thus expressed in terms of the hypergeometric series by the same formula (8), but with θ replaced by 2θ.

From (7), (8) and (9a) follows

$$H_{2n}(x|q)=(-q^{\frac{1}{2}},q^{\frac{1}{2}})_{2n}q^{-n/2}\,{}_3\phi_2\left(\begin{matrix}q^{-n},q^{\frac{1}{2}}\mathrm{e}^{2\mathrm{i}\theta},q^{\frac{1}{2}}\mathrm{e}^{-2\mathrm{i}\theta}\\ -q,-q^{\frac{1}{2}}\end{matrix}\middle|q;q\right). \tag{14}$$

The transformation of a terminating series

$${}_3\phi_2\left(\begin{matrix}q^{-n},b,c\\ d,e\end{matrix}\middle|q;q\right)=\frac{(de/bc;q)_n}{(e;q)_n}\left(\frac{bc}{d}\right)^n{}_3\phi_2\left(\begin{matrix}q^{-n},d/b,d/c\\ d,de/bc\end{matrix}\middle|q;q\right) \tag{15}$$

enables (14) to be rewritten as

$$H_{2n}(x|q)=(-1)^n(q;q^2)_n\,{}_3\phi_2\left(\begin{matrix}q^{-n},-\mathrm{e}^{2\mathrm{i}\theta},-\mathrm{e}^{-2\mathrm{i}\theta}\\ q^{1/2},-q^{1/2}\end{matrix}\middle|q;q\right). \tag{16}$$

It remains to apply Singh's quadratic transformation for a terminating ${}_3\phi_2$ series

$${}_3\phi_2\left(\begin{matrix}a^2,b^2,c\\ abq^{\frac{1}{2}},-abq^{\frac{1}{2}}\end{matrix}\middle|q;q\right)={}_3\phi_2\left(\begin{matrix}a^2,b^2,c^2\\ a^2b^2q,0\end{matrix}\middle|q^2;q^2\right) \tag{17}$$

from base q to base q^2. This gives (cf [10])

$$H_{2n}(x|q)=(-1)^n(q;q^2)_n\,{}_3\phi_2\left(\begin{matrix}q^{-2n},-\mathrm{e}^{2\mathrm{i}\theta},-\mathrm{e}^{-2\mathrm{i}\theta}\\ q,0\end{matrix}\middle|q^2;q^2\right). \tag{18}$$

In a like manner,

$$H_{2n+1}(x|q)=(-q)^{-n}(q^3;q^2)_n2x\,{}_3\phi_2\left(\begin{matrix}q^{-2n},-q\mathrm{e}^{2\mathrm{i}\theta},-q\mathrm{e}^{-2\mathrm{i}\theta}\\ q^3,0\end{matrix}\middle|q^2;q^2\right). \tag{19}$$

Now we are in a position to express (1) as a sum of two ${}_2\phi_1$'s. Indeed, separating odd and even powers of t, we find

$$I(x,t;q)=\sum_{n=0}^{\infty}\frac{q^{n^2}t^{2n}}{(q;q)_{2n}}H_{2n}(x|q)+q^{\frac{1}{4}}\sum_{n=0}^{\infty}\frac{q^{n(n+1)}t^{2n+1}}{(q;q)_{2n+1}}H_{2n+1}(x|q). \tag{20}$$

Next substituting (18) into the first sum in (20) and taking into account that $(q;q)_{2n}=(q,q^2;q^2)_n$, defines

$$\begin{aligned}I_1(\sin\gamma,t;q)&=\sum_{n=0}^{\infty}\frac{q^{n^2}t^{2n}}{(q;q)_{2n}}H_{2n}(\sin\gamma|q)\\ &=\sum_{n=0}^{\infty}\frac{q^{n^2}(-t^2)^n}{(q^2;q^2)_n}\sum_{k=0}^{n}\frac{(q^{-2n},\mathrm{e}^{2\mathrm{i}\gamma},\mathrm{e}^{-2\mathrm{i}\gamma};q)_k}{(q,q^2;q^2)_k}q^{2k}.\end{aligned} \tag{21}$$

Note that in (21) and all subsequent formulae we consider the parametrization $x = \sin\gamma$, $\gamma = \frac{1}{2}\pi - \theta$ to be more convenient than $x = \cos\theta$ (cf formulae (2) and (3) above). Interchanging the order of summation in (21), the relation (easily verified by induction)

$$\frac{(q^{-2(n+k)};q^2)_k}{(q^2;q^2)_{n+k}} = \frac{(-1)^k}{(q^2;q^2)_n} q^{-2nk-k(k+1)} \tag{22}$$

gives

$$I_1(\sin\gamma, t; q) = \sum_{k=0}^{\infty} \frac{(\mathrm{e}^{2\mathrm{i}\gamma}, \mathrm{e}^{-2\mathrm{i}\gamma}; q^2)_k}{(q, q^2; q^2)_k} (qt^2)^k \sum_{n=0}^{\infty} \frac{q^{n^2}(-t^2)^n}{(q^2;q^2)_n}. \tag{23}$$

The sum over n does not depend on the variable $x = \sin\gamma$ and is readily recognized as the q-exponential function (5′). The sum over k yields a ${}_2\phi_1$ basic hypergeometric series (see (10)). Thus

$$I_1(\sin\gamma, t; q) = E_{q^2}(-t^2)\, {}_2\phi_1(q\mathrm{e}^{2\mathrm{i}\gamma}, q\mathrm{e}^{-2\mathrm{i}\gamma}; q; q^2, t^2). \tag{24}$$

Similarly, substituting (19) into the second term in (20), we obtain

$$I_2(\sin\gamma, t; q) = \frac{2q^{\frac{1}{4}}t}{1-q} E_{q^2}(-t^2) \sin\gamma\, {}_2\phi_1(q^2\mathrm{e}^{2\mathrm{i}\gamma}, q^2\mathrm{e}^{-2\mathrm{i}\gamma}; q^3; q^2, t^2). \tag{25}$$

The sum of (24) and (25) yields the following expression for (1) in terms of the ${}_2\phi_1$ functions:

$$I(\sin\gamma, t; q) = E_{q^2}(-t^2)\Big\{ {}_2\phi_1(q\mathrm{e}^{2\mathrm{i}\gamma}, q\mathrm{e}^{-2\mathrm{i}\gamma}; q; q^2, t^2) + \frac{2q^{\frac{1}{4}}t}{1-q} \sin\gamma\, {}_2\phi_1(q^2\mathrm{e}^{2\mathrm{i}\gamma}, q^2\mathrm{e}^{-2\mathrm{i}\gamma}; q^3; q^2, t^2)\Big\}. \tag{26}$$

If one thus defines the q-exponential function $\varepsilon_q(x;\tau)$ (see [6]) via

$$\sum_{n=0}^{\infty} \frac{q^{n^2/4}t^n}{(q;q)_n} H_n(x|q) = E_{q^2}(-t^2)\varepsilon_q(x; -\mathrm{i}t) \tag{27}$$

then from (1) and (26) follows

$$\varepsilon_q(\sin\gamma;\tau) = {}_2\phi_1(q\mathrm{e}^{2\mathrm{i}\gamma}, q\mathrm{e}^{-2\mathrm{i}\gamma}; q; q^2, -\tau^2) + \frac{2\mathrm{i}q^{\frac{1}{4}}\tau}{1-q} \sin\gamma\, {}_2\phi_1(q^2\mathrm{e}^{2\mathrm{i}\gamma}, q^2\mathrm{e}^{-2\mathrm{i}\gamma}; q^3; q^2, -\tau^2). \tag{28}$$

Another form of the $\varepsilon_q(\sin\gamma;\tau)$-function, equivalent to (28), is

$$\varepsilon_q(\sin\gamma;\tau) = \sum_{n=0}^{\infty} \frac{\tau^n}{(q;q)_n} q^{n^2/4} (q^{(1-n)/2}\mathrm{e}^{-\mathrm{i}\gamma}, -q^{(1-n)/2}\mathrm{e}^{\mathrm{i}\gamma}; q)_n. \tag{29}$$

By definition (27), the function $\varepsilon_q(x;\tau)$ satisfies the difference equation (cf difference equations (4.5.1.3) and (4.5.2.2) in [11] for the q-exponential functions $e_q(x)$ and $E_q(x)$, respectively)

$$\sin\kappa\partial_x\, \varepsilon_q(\sin\kappa x;\tau) = \mathrm{i}q^{-\frac{1}{4}}\tau \cos\kappa x\, \varepsilon_q(\sin\kappa x;\tau). \tag{30}$$

This can be readily verified by applying the operator $\sin\kappa\partial_x$ to both sides of (27) and using the fact that it acts as the lowering operator on the continuous q-Hermite polynomials, that is

$$\sin\kappa\partial_x\, H_n(\sin\kappa x|q) = (q^{-n/2} - q^{n/2}) \cos\kappa x\, H_{n-1}(\sin\kappa x|q). \tag{31}$$

It turns out that the function $\varepsilon_q(x;\tau)$ can be expressed as a Fourier–Gauss transform of the product of two q-exponential functions (5′). Indeed, the continuous q-Hermite $H_n(x|q)$ and q^{-1}-Hermite $h_n(x|q)$ polynomials are known to be related to each other by the Fourier–Gauss transform [12]

$$H_n(\sin\kappa x|q)\mathrm{e}^{-x^2/2} = \frac{\mathrm{i}^n}{\sqrt{2\pi}} q^{n^2/4} \int_{-\infty}^{\infty} h_n(\sinh\kappa y)\mathrm{e}^{-\mathrm{i}xy-y^2/2}\,\mathrm{d}y. \tag{32}$$

Substituting (32) into the left-hand side of (27) and summing over n by the aid of the generating function [13, 14]

$$\sum_{n=0}^{\infty} \frac{t^n q^{n(n-1)/2}}{(q;q)_n} h_n(\sinh\kappa x|q) = E_q(t\mathrm{e}^{\kappa x})E_q(-t\mathrm{e}^{-\kappa x}) \tag{33}$$

for the q^{-1}-Hermite polynomials $h_n(x|q)$, one obtains

$$\varepsilon_q(\sin\kappa x;\tau)E_{q^2}(\tau^2)\mathrm{e}^{-x^2/2} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}xy-y^2/2} E_q(\tau q^{\frac{1}{2}}\mathrm{e}^{-\kappa y})E_q(-\tau q^{\frac{1}{2}}\mathrm{e}^{\kappa y})\,\mathrm{d}y. \tag{34}$$

It is interesting to compare (34) with Ramanujan's integral [15–17]

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} E_q(aq^{\frac{1}{2}}\mathrm{e}^{\sqrt{2}\kappa y})E_q(bq^{\frac{1}{2}}\mathrm{e}^{-\sqrt{2}\kappa y})\mathrm{e}^{-\mathrm{i}xy-y^2/2}\,\mathrm{d}y$$
$$= E_q(-ab)e_q(a\mathrm{e}^{-\mathrm{i}\sqrt{2}\kappa x})e_q(b\mathrm{e}^{\mathrm{i}\sqrt{2}\kappa y})\mathrm{e}^{-x^2/2}. \tag{35}$$

In (34), equating parts symmetric and antisymmetric (respectively) with respect to the parameter τ yields the integral representations

$$ {}_2\phi_1(q\mathrm{e}^{2\mathrm{i}\kappa x}, q\mathrm{e}^{-2\mathrm{i}\kappa x}; q; q^2, -\tau^2)E_{q^2}(\tau^2)\mathrm{e}^{-x^2/2}$$
$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} E_q(\tau q^{\frac{1}{2}}\mathrm{e}^{-\kappa y})E_q(-\tau q^{\frac{1}{2}}\mathrm{e}^{\kappa y})\mathrm{e}^{-y^2/2}\cos xy\,\mathrm{d}y \tag{36a}$$

$$\tau\sin\kappa x\, {}_2\phi_1(q^2\mathrm{e}^{2\mathrm{i}\kappa x}, q^2\mathrm{e}^{-2\mathrm{i}\kappa x}; q^3; q^2, -\tau^2)E_{q^2}(\tau^2)\mathrm{e}^{-x^2/2}$$
$$= \frac{1-q}{2\sqrt{2\pi}} q^{-\frac{1}{4}} \int_{-\infty}^{\infty} E_q(\tau q^{\frac{1}{2}}\mathrm{e}^{\kappa y})E_q(-\tau q^{\frac{1}{2}}\mathrm{e}^{-\kappa y})\mathrm{e}^{-y^2/2}\sin xy\,\mathrm{d}y \tag{36b}$$

for the non-terminating basic hypergeometric series ${}_2\phi_1$ under consideration.

Combining the Ramanujan-type orthogonality relation for the continuous q-Hermite polynomials [12]

$$\int_{-\infty}^{\infty} H_m(\sin\kappa x|q)H_n(\sin\kappa x|q)\mathrm{e}^{-x^2}\cos\kappa x\,\mathrm{d}x = \sqrt{\pi}q^{\frac{1}{8}}(q;q)_m\delta_{mn} \tag{37}$$

with the definition (27) of the q-exponential function $\varepsilon_q(x;t)$, yields the following two integrals

$$\int_{-\infty}^{\infty} H_m(\sin\kappa x|q)\varepsilon_q(\sin\kappa x;t)\mathrm{e}^{-x^2}\cos\kappa x\,\mathrm{d}x = \sqrt{\pi}q^{\frac{1}{4}(m^2+\frac{1}{2})}(\mathrm{i}t)^m\varepsilon_{q^2}(-t^2) \tag{38}$$

$$\int_{-\infty}^{\infty} \varepsilon_q(\sin\kappa x;t)\varepsilon_q(\sin\kappa x;\tau)\mathrm{e}^{-x^2}\cos\kappa x\,\mathrm{d}x = \sqrt{\pi}q^{\frac{1}{8}}e_{q^2}(-t^2)e_{q^2}(-\tau^2)E_q(-q^{\frac{1}{2}}t\tau). \tag{39}$$

We note in closing that the Fourier–Gauss transformation properties of the continuous q-Hermite polynomials and the corresponding coherent states, discussed above, are closely related to their convolution properties. We hope to discuss this question in detail elsewhere.

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